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Iterative evaluation of the complex constants of piezoceramic resonators in the thickness mode

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Abstract

An iterative method is presented for the characterization of lossy piezoelectric materials in the thickness resonant mode, which allows to calculate the frequencies at which the electrical impedance is to be measured. This new method considerably reduces the measurement time by separating the programs for data acquisition and for processing thus simplifying the measurement procedure. The accuracy of the method was tested for several piezoceramic materials, covering a wide range of values of the thickness coupling factor and mechanical quality factor. The new method proved to be as accurate as other iterative methods, but it is easier to apply.

Keywords: Impedance; Piezoelectric properties; Testing

1. Introduction

In a previous paper,[1] we proposed a new improved iterative method to determine the material constants, in complex form, for piezoceramic resonators in the radial mode. This new method provided a new formula to calculate the frequencies at which the admittance is measured. Its main advantage consisted in reducing the measurement time, by separating the programs for data acquisition and processing. The method was tested on several representative piezoceramic materials, in the radial mode and it was compared with some other similar methods.[2,3] The results showed similar accuracy, but our method proved to be more rapid and easier to apply and even more accurate for materials with large coupling factor.

These facts encouraged us to extend this method for the thickness extensional mode too, since many piezoelectric sensors and actuators use this vibration mode. Therefore, the present paper reports on the results obtained in determining the material constants for the thickness mode. These results were comparatively discussed with those obtained by similar methods.[4]

2. Measurements

The measurement technique consists in generating a thickness extensional mode of vibration in a disc shaped piezoceramic resonator, by sinusoidal electrical stimulation and frequency sweep. This was done by means of an HP-4194A impedance gain/phase analyzer, controlled by a computer.

The real (resistance R) and imaginary (reactance X) parts of the complex electrical impedance Z were measured as a function of frequency within the range of resonance and antiresonance of the fundamental thickness mode. These data were stored as input resonance spectrum, in order to check only the agreement with the output data calculated with the constants provided by this method.

The series and parallel resonance frequencies f_s and f_p corresponding to the maxima of G (conductance) and R, respectively, were also determined and stored together with their corresponding values of complex impedances Z_s and Z_p. The frequencies $f_{X\max}$ and $f_{X\min}$ of maximum and minimum of X around f_p were then determined and stored. The frequencies $f_{1,2\mathrm{eff}}$ were calculated with the formula:

$$f_{1,2\mathrm{eff}} = \frac{f_s}{\sqrt{1 \pm k_{\mathrm{teff}}}} \quad (1)$$

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where k_{teff} is given by:

$$k_{teff} = \sqrt{\frac{f_p^2 - f_s^2}{f_p^2}} \quad (2)$$

The complex values $Z_{1,2eff}$ of the electrical impedance, measured at these frequencies, were also stored.

The method requires that the resonance spectrum should be free of spurious modes, therefore the radial and thickness modes should be completely separated, by choosing appropriate dimensions of the resonator.

As one can see, the frequencies $f_{1,2eff}$ at which the impedance has to be measured are only determined as a function of f_s and f_p, without requiring any result of the iterative procedure (used for determining the material constants). The other frequencies required by this method: f_s, f_p, $f_{X\max}$ and $f_{X\min}$ are determined directly from the resonance spectrum. Thus the measurements and the processing are completely independent. The separation between data acquisition and processing considerably reduces the measurement time to a few seconds and simplify the experimental procedure. This is the main difference between our method and Alemany's,[4] which requires new measurements of the admittance during the iteration process.

3. Description of the method

The method follows the same algorithm as the previous one,[1] but it uses the measurement of electrical impedance instead of electrical admittance to determine the material constants of piezoceramic resonators in the thickness extensional mode. The electrical impedance Z, of such resonators, as a function of the oscillating frequency f is given by:[5]

$$Z = -i\frac{t}{2\pi f A \varepsilon_{33}^S} + i\frac{h_{33}^2 \tan\left(\pi f t\sqrt{\rho/c_{33}^D}\right)}{2\pi^2 A f^2 c_{33}^D \sqrt{\rho/c_{33}^D}} \quad (3)$$

where A, t and ρ are the electrode area, thickness and density of the sample, respectively and $i = \sqrt{-1}$. The dielectric permittivity at constant strain ε_{33}^S, the piezoelectric constant h_{33}, and the elastic stiffness at constant dielectric displacement c_{33}^D are complex quantities.

The electrical impedance Z is measured at three different frequencies f_1, f_2 and f_3, in order to determine the three material constants ε_{33}^S, h_{33} and c_{33}^D from Eq. (3). This is accomplished by an iterative process that requires initial estimation of the elastic constant c_{33}^D, given by:

$$c_{33in}^D = 4\rho t^2 f_p^2 \left[1 + i\frac{f_{X\min} - f_{X\max}}{f_p}\right] \quad (4)$$

The real part of c_{33in}^D is the value provided by IEEE Std.,[5] for a lossless resonator.

With this initial estimation of c_{33}^D the electrical impedance from (3) becomes linear in $1/\varepsilon_{33}^S$ and h_{33}^2. Measuring Z at frequencies f_1 and f_2, a linear system of two equations is obtained. By solving it, ε_{33}^S and h_{33}^2 are then determined. Next, a better approximation of c_{33}^D is obtained from the argument of the tangent function, calculated from Eq. (3) after substituting the immediately preceding values of ε_{33}^S, h_{33} and c_{33}^D and the experimental value of Z measured at f_3. Care must be taken to choose the arguments of negative tangents in the second quadrant, since the fundamental resonance corresponds to angles next to $\pi/2$. One step in the iteration process is complete now and it will be repeated until the cut-off criterion:

$$\frac{\left|c_{33f}^D - c_{33i}^D\right|}{\left|c_{33f}^D\right|} \leqslant 10^{-8} \quad (5)$$

is fulfilled. The subscripts "i" and "f" correspond to the initial and final values of c_{33}^D in an iteration step.

The frequencies f_1, f_2 and f_3 were carefully selected, since they drastically influence the accuracy of this method. Thus, $f_{1,2eff}$ were chosen to substitute for $f_{1,2}$ and f_p for f_3. The elastic constant c_{33}^D was determined at

Table 1
The values of the input constants for materials **A**, **B**, **C**, **D** and **E**, respectively

Input constants	Materials				
	A	**B**	**C**	**D**	**E**
$\varepsilon_{33}^S/\varepsilon_0$	700−i 5	400−i	125−i 3	210−i 5	235−i 5
h_{33} (10^8 Vm^{-1})	30+i 0.3	38+i 0.1	45+i	21+i	10+i
c_{33}^D (10^{10} Nm^{-2})	15+i 0.15	15+i 0.03	15+i 0.05	5+i 0.25	5+i 0.25
k_t	0.61+i 0.00087	0.58+i 0.00022	0.39+i 0.0033	0.40+i 0.0043	0.20+i 0.013
k_{teff}	0.57	0.54	0.35	0.37	0.18
Q_m	100	500	300	20	20
$f_p/(f_{X\min} - f_{X\max})$	100	500	300	20	21
ρ (kg/m^3)	7650	7600	7550	6000	6000

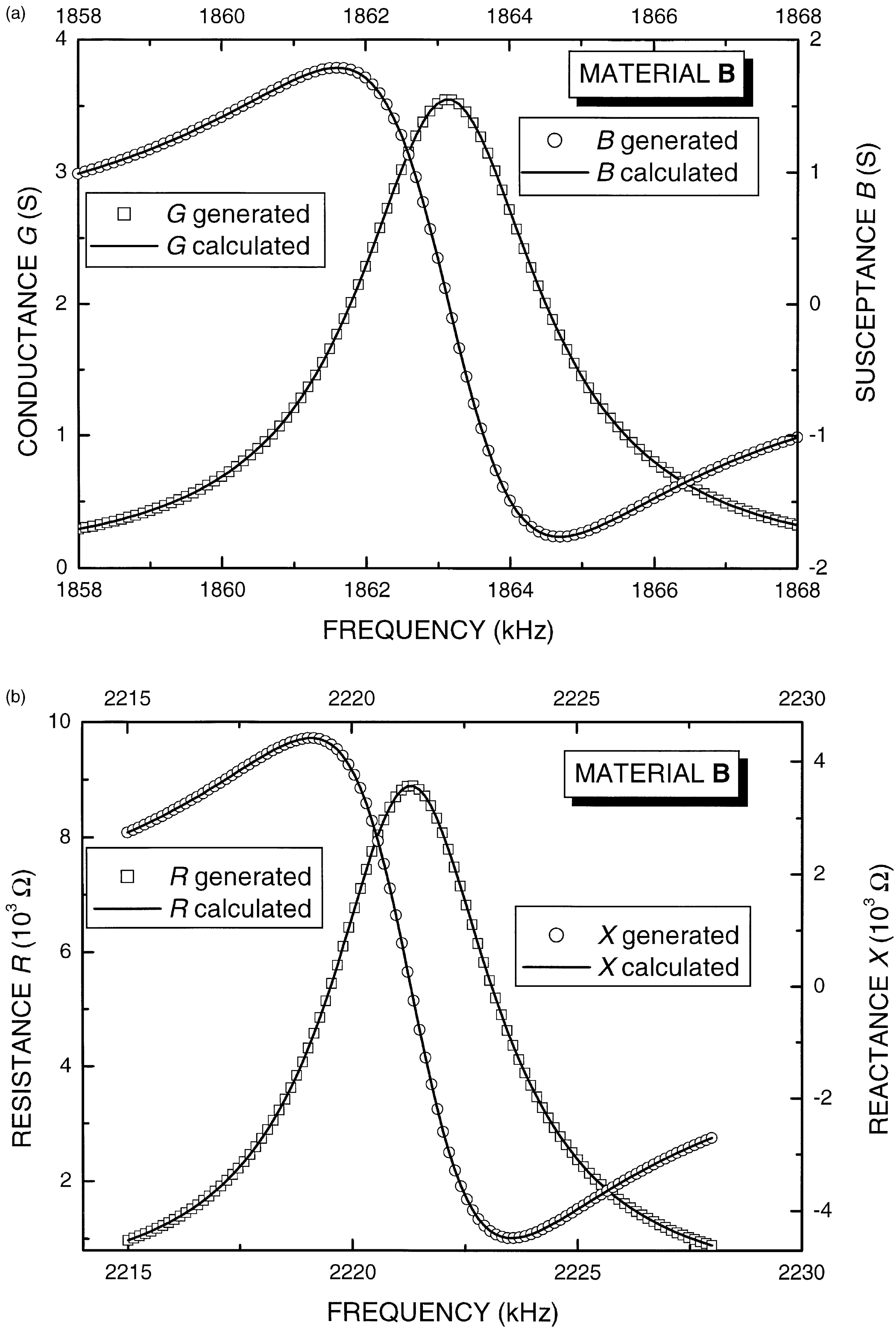

Fig. 1. Resonance (a) and antiresonance (b) generated and calculated spectra of the fundamental thickness mode for material **B**.

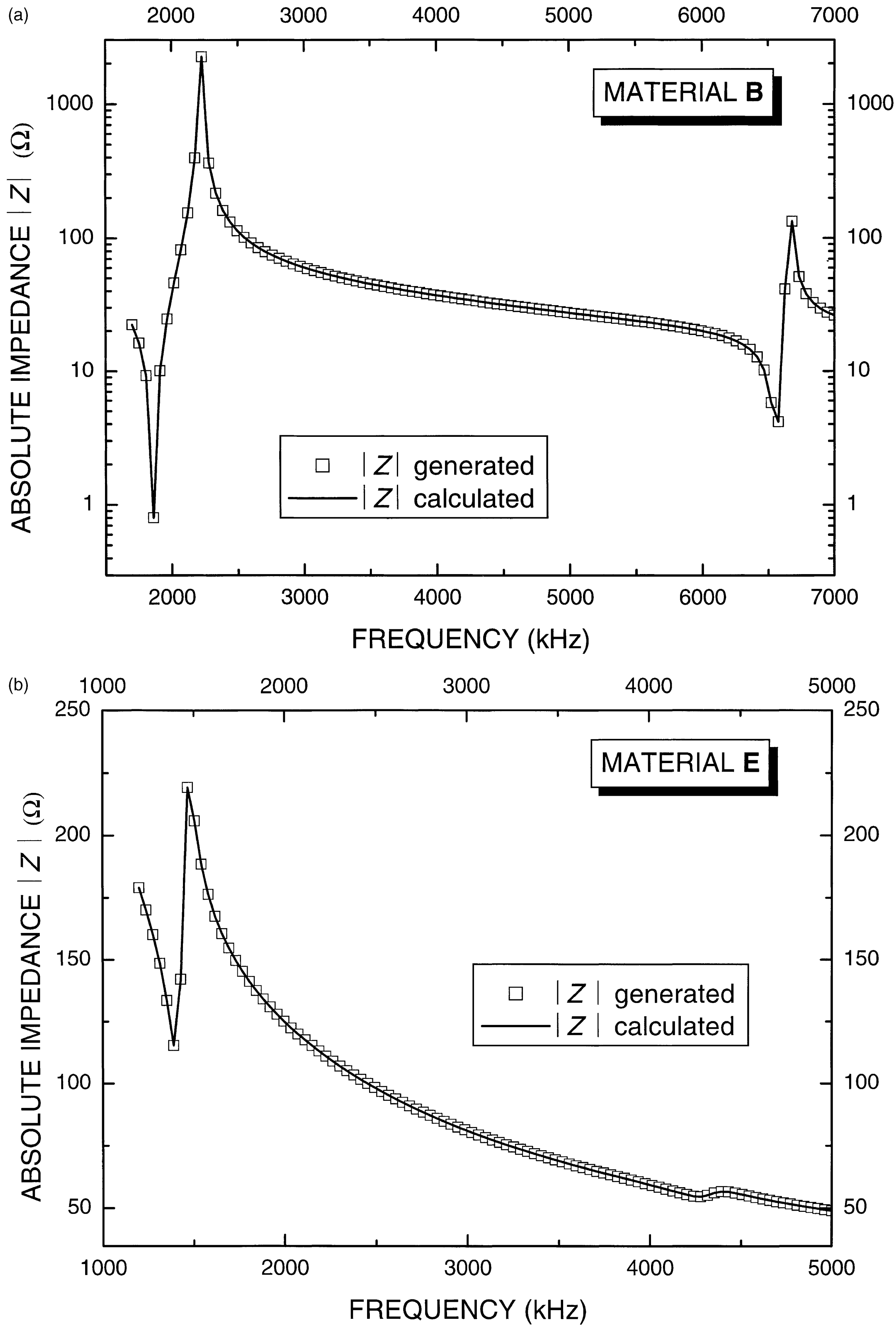

Fig. 2. (a) Resonance and (b) antiresonance generated and calculated spectra of the fundamental thickness mode for material **B**.

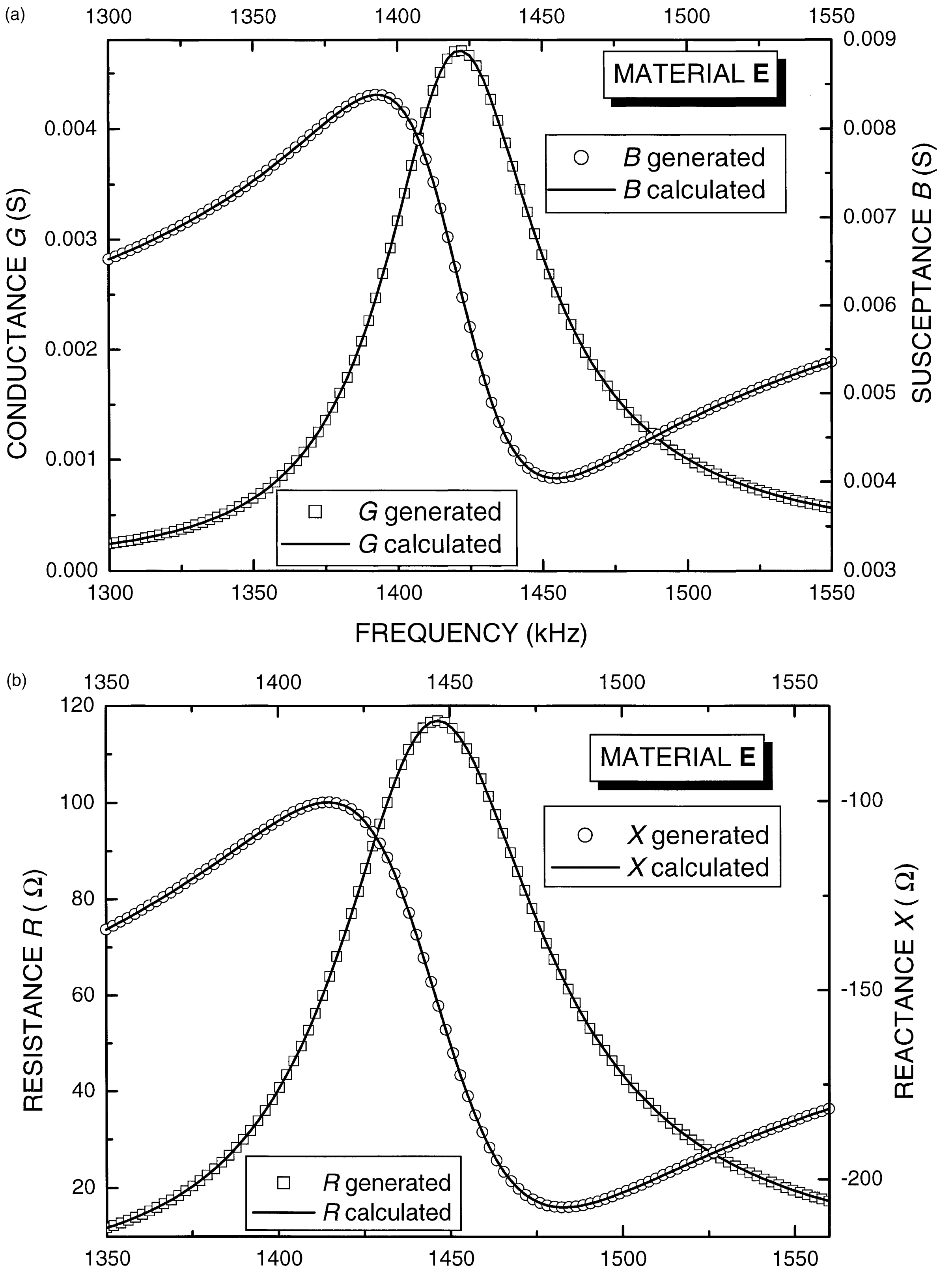

Fig. 3. Absolute generated and calculated impedance as a function of frequency, in the range of the fundamental resonance and the first overtone of the thickness mode for materials **B** (fig. a) and **E** (fig. b) respectively.

f_p, since the electrical parallel resonance of this mode corresponds to mechanical resonance, where a large amount of input energy is converted into elastic energy. This allows an accurate determination of the elastic constant, while a poor accuracy will be expected for the dielectric constant, since the dielectric energy is small compared to the elastic energy. Therefore, it is indicated to determine the dielectric constant far from f_p, at $f_{1,2\mathrm{eff}}$ where the dielectric energy becomes dominant.

The algorithm of this method was written in Mathematica 3.0.

4. Results and discussion

In order to include all kind of piezoceramic materials used for applications, this method was tested on five representative materials, with different characteristics: material **A** (large k_t and moderate Q_m), material **B** (large k_t and high Q_m), material **C** (moderate k_t and high Q_m), material **D** (moderate k_t and low Q_m) and material **E** (small k_t and low Q_m). They cover the whole range of values for the thickness coupling factors (k_t = 20–61%) and mechanical quality factors (Q_m = 20–500). The electromechanical behavior of each material was simulated by giving complex values for material constants ε_{33}^S, h_{33} and c_{33}^D, considered as input constants, and then by generating the impedance data using Eq. (3). The method was applied using the generated impedance data, as experimental (input) resonance spectrum, in order to calculate the material constants and to compare them with their input values. The accuracy in determining the real and imaginary parts of each material constant was estimated by the following relationships:

$$\varepsilon_{\mathrm{const}'} = \frac{|\mathrm{const}'_i - \mathrm{const}'_c|}{|\mathrm{const}'_i|} \tag{6}$$

and

$$\varepsilon_{\mathrm{const}''} = \frac{|\mathrm{const}''_i - \mathrm{const}''_c|}{|\mathrm{const}''_i|} \tag{7}$$

where subscripts "i" and "c" designate the input and calculated constants, respectively and single and double primes signify the real and imaginary parts, respectively.

The resonance spectra were generated for disc shaped resonators of 20 mm in diameter and 1mm thickness, by using a program in Mathematica 3.0.

Table 1 shows the input constants for materials **A**–**E** and input k_t and Q_m calculated with input constants by the following formulas:

$$k_t = h_{33}\sqrt{\frac{\varepsilon_{33}^S}{c_{33}^D}} \tag{8}$$

$$Q_m = -\frac{c_{11}^{p'}}{c_{11}^{p''}} \tag{9}$$

The initial estimation of c_{33}^D from (4) is in agreement with (9), since the values of input Q_m and $f_p/(f_{X\min} - f_{X\max})$ ratio are very close (see Table 1).

The material constants determined by this method were substituted in Eq. (3) to calculate the output impedance data, in the range of resonance–antiresonance of fundamental thickness extensional mode, in order to check their agreement with the generated impedance data.

As an exemple, we selected two materials, **B** and **E**, with very different electromechanical behavior, to prove the fit with the generated data. Figs. 1 and 2 show the real and imaginary parts of the generated and calculated admittance and impedance, as a function of frequency, in the range of resonance and antiresonance of the fundamental thickness mode, for materials **B** and **E**, respectively. Fig. 3 shows the absolute generated and calculated impedance as a function of frequency, in the range of the fundamental resonance and first overtone of the thickness mode, for the same materials. One can see that output and input data are in very good agreement, for both materials. Similar results were also obtained for **A**, **C** and **D** materials.

Table 2 shows the accuracy of the method, for all investigated materials. The coupling factor k_t, the piezoelectric constant e_{33} and the elastic stiffness c_{33}^E, at constant electric field, were determined with the previous constants, from the Eq. (8) and the following ones:

$$e_{33} = h_{33}\varepsilon_{33}^S \tag{10}$$

$$c_{33}^E = c_{33}^D - h_{33}^2\varepsilon_{33}^S \tag{11}$$

Table 2
The accuracy of the method for materials **A**, **B**, **C**, **D** and **E**, respectively

Constant		Accuracy (%)				
		A	**B**	**C**	**D**	**E**
$\varepsilon_{33}^S/\varepsilon_0$	Real	2×10^{-7}	6×10^{-9}	2×10^{-8}	10^{-7}	2×10^{-7}
	Imag.	3×10^{-6}	2×10^{-4}	2×10^{-7}	5×10^{-6}	4×10^{-5}
h_{33}	Real	6×10^{-10}	3×10^{-9}	7×10^{-11}	4×10^{-8}	10^{-6}
	Imag.	6×10^{-7}	10^{-6}	2×10^{-8}	7×10^{-7}	2×10^{-6}
c_{33}^D	Real	2×10^{-10}	4×10^{-10}	10^{-11}	10^{-9}	8×10^{-9}
	Imag.	10^{-7}	8×10^{-9}	10^{-8}	8×10^{-8}	10^{-6}
k_t	Real	10^{-7}	10^{-9}	10^{-8}	5×10^{-9}	10^{-6}
	Imag.	4×10^{-6}	6×10^{-4}	2×10^{-7}	8×10^{-6}	8×10^{-6}
c_{33}^E	Real	10^{-7}	2×10^{-9}	3×10^{-9}	5×10^{-9}	7×10^{-8}
	Imag.	10^{-6}	2×10^{-4}	2×10^{-6}	6×10^{-7}	2×10^{-8}
e_{33}	Real	2×10^{-7}	5×10^{-9}	2×10^{-8}	5×10^{-8}	10^{-6}
	Imag.	6×10^{-6}	3×10^{-3}	2×10^{-6}	6×10^{-6}	10^{-5}

As one can see, the method provided very high accuracy for all constants of each material (even for materials **D** and **E** with high losses). The accuracy of our method was similar to that of other iterative methods.[4]

The main advantage of this method consists in its simple and rapid technique of measurement, completely separated from the program for determining the material constants, which may be very useful for piezoceramics characterization.

5. Conclusions

An improved iterative method, for determining the dielectric, piezoelectric and elastic constants, in complex form, for the piezoceramic materials, in the thickness mode, was proposed. It is based on the measurement of the electrical impedance at three properly chosen frequencies, determined by the program for data acquisition. Two of these frequencies are calculated by a new simple relationship, provided by this method, without requiring intermediate results of the iterative process. This allows the separation between the data acquisition and processing, thus reducing the measurement time. This separation is possible, since the new method implies no further measurements during the calculation of the material constants and this is the main benefit of it.

The new method was tested for several representative materials, covering a large spectrum of piezoelectric properties, and it proved to be very accurate and easy to apply. Therefore, one may conclude that it may be used for any piezoceramic material characterization.

References

1. Amarande, L., Miclea, C. and Tanasoiu, C., Iterative evaluation of the complex constants of piezoceramic resonators in the radial mode. *J. Eur. Ceram. Soc*. 2002, **22**, 1873–1881.
2. Smits, J. G., Iterative method for accurate determination of the real and imaginary parts of the material coefficients of piezoelectric ceramics. *IEEE Trans. on Sonics and Ultrasonics*, 1976, **SU-23**(6), 393–402.
3. Alemany, C., Gonzales, A. M., Pardo, L., Jimenez, B., Carmona, F. and Mendiola, J., Automatic determination of complex constants of piezoelectric lossy materials in the radial mode. *J. Phys. D: Appl. Phys*., 1995, **28**, 945–956.
4. Alemany, C., Pardo, L., Jimenez, B., Carmona, F., Mendiola, J. and Gonzales, A. M., Automatic iterative evaluation of complex material constants in piezoelectric ceramics. *J. Phys. D: Appl. Phys*., 1994, **27**, 148–155.
5. *IEEE Standard on piezoelectricity, 176–1987*, ANSI-IEEE Std.176, 1987.